

# ON APPROXIMATE STRUCTURE OF THE TIME-OPTIMAL SYSTEM DESIGN ALGORITHM

*A. Zemliak, E. Rios*

Puebla Autonomous University  
azemliak@cfm.buap.mx

## ABSTRACT

The process of analog system design has been formulated on the basis of the control theory application. This approach generalizes the design process and produces the different design trajectories inside the same optimization procedure. Numerical examples show that the potential computer time gain of the optimal design strategy with respect to the traditional design strategy increases when the size and complexity of the system increase. However the potential time gain can be realized only in the time-optimal algorithm that is the combination of the different design strategies. An additional acceleration effect of the design process serves as the basis of the time-optimal algorithm construction. The optimal position of the switching points between different design trajectories can be obtained on the basis of the Lyapunov function of the design process as the minimization of its time derivative.

## RESUMEN

El proceso de diseño de un sistema análogo ha sido formulado en la base de aplicación de la teoría de control. Este enfoque generaliza el proceso de diseño y produce las trayectorias distintas dentro del mismo procedimiento de optimización. Ejemplos numéricos muestran que la ganancia potencial del tiempo de cómputo de una estrategia óptima en comparación con la estrategia tradicional crece cuando el tamaño y la complejidad del sistema crecen. Sin embargo la ganancia potencial se realiza solo en caso cuando el algoritmo óptimo ha sido construido como la combinación de varias estrategias de diseño. Un efecto de aceleración adicional del proceso de diseño sirve como la base de la construcción del algoritmo óptimo en el tiempo. Las posiciones óptimas de los puntos de conmutación entre diferentes trayectorias de diseño se pueden obtener en la base de la función de Lyapunov del proceso de diseño por medio de la minimización de la derivada temporal de esta función.

# ON APPROXIMATE STRUCTURE OF THE TIME-OPTIMAL SYSTEM DESIGN ALGORITHM

*A. Zemliak, E. Rios*

Puebla Autonomous University

## ABSTRACT

The process of analog system design has been formulated on the basis of the control theory application. This approach generalizes the design process and produces the different design trajectories inside the same optimization procedure. Numerical examples show that the potential computer time gain of the optimal design strategy with respect to the traditional design strategy increases when the size and complexity of the system increase. However the potential time gain can be realized only in the time-optimal algorithm that is the combination of the different design strategies. An additional acceleration effect of the design process serves as the basis of the time-optimal algorithm construction. The optimal position of the switching points between different design trajectories can be obtained on the basis of the Lyapunov function of the design process as the minimization of its time derivative.

## 1. INTRODUCTION

The problem of the computer time reduction of a large system design is one of the essential problems of the total quality design improvement. There are some powerful methods that reduce the necessary time for the circuit analysis by means of the special sparse matrix techniques [1]-[2] or by the partitioning of a circuit matrix [3]-[4]. However there is another way to solve this problem. The generalized approach for the system design on the basis of control theory formulation was elaborated in some previous works [1]-[2]. This approach serves for the time-optimal design algorithm definition. On the other hand this approach gives the possibility to analyze with a great clearness the design process while moving along the trajectory curve into the design space. The main conception of the theory is the introduction of the special control functions, which, on the one hand generalize the design process and, on the other hand, they give the possibility to control design process to achieve the optimum of the design objective function for the minimum computer time. This possibility appears because practically an infinite number of the different design strategies that exist within the bounds of the theory, but the different design strategies have the different operation

number and executed computer time. On the bounds of this conception, the traditional design strategy is only a one representative of the enormous set of different design strategies. As shown in [5] the potential computer time gain that can be obtained by the new design problem formulation increases when the size and complexity of the system increase but it is realized only in case when we have the algorithm for the optimal trajectories systematical construction. We can define the formulation of the intrinsic properties and special restrictions of the optimal design trajectory as one of the first problems that needs to be solved for the optimal algorithm construction.

## 2. PROBLEM FORMULATION

The design process for any analog system design is defined as the problem of the objective function  $F(X)$  minimization for  $X \in R^N$  with the system of constraints that are the mathematical model of the designed system. The parametric optimization process for the objective function  $F(X)$  minimization for two-step procedure is defined in general case as following vector equation:

$$X^{s+1} = X^s + t_s \cdot H^s \quad (1)$$

with constraints (1), where  $s$  is the iterations number,  $t_s$  is the iteration parameter,  $t_s \in R^1$ ,  $H$  is the direction of the objective function  $F(X)$  decreasing. We suppose that the mathematical model of the designed can be described by the nonlinear algebraic equations:

$$g_j(X) = 0 \quad (2)$$

$$j = 1, 2, \dots, M$$

The main idea of the previous analysis [5]-[6] is the reformulation of the problem (1), (2) on the basis of the control theory. In this case the design process is defined by means of the optimization procedure and can be

written in the vector form as:

$$\frac{dX}{dt} = f(X, U) \quad (3)$$

or in coordinate form as:

$$\frac{dx_i}{dt} = f_i(X, U) \quad (3')$$

$$i=1,2,\dots,K,K+1,\dots,N$$

with an additional nonlinear system:

$$(1 - u_j) g_j(X) = 0 \quad (4)$$

$$j = 1, 2, \dots, M$$

Equations (3) or (3') describe the design process in continuous form. This process can be described in the discrete form as:

$$x_i^{s+1} = x_i^s + t_s \cdot f_i(X, U) \quad (5)$$

$$i=1,2,\dots,K,K+1,\dots,N$$

A special vector  $U = (u_1, u_2, \dots, u_m)$ , where  $u_j \in \Omega$ ;  $\Omega = \{0;1\}$  is defined for generalization of the design problem. The components of the vector  $U$  have a sense as the control functions of the design process [6]. The function  $f(X, U)$  is the directional movement vector  $H$  and has dependency from the generalized objective function  $F(X, U)$ . It means that the main problem of the design process can be formulated as the problem of the integration of this system (3) with additional conditions (4). The structure of the function  $H$  for three different orders optimization methods can be defined as:

$$H \equiv f(F(X, U)) = -F'(X, U) \quad (6)$$

for the gradient method,

$$H \equiv f(F(X, U)) = -\{F''(X, U)\}^{-1} \cdot F'(X, U) \quad (6')$$

for the Newton's method, where  $F''(X, U)$  is a matrix of second derivatives,

$$H \equiv f(F(X, U)) = -B(X, U) \cdot F'(X, U) \quad (6'')$$

for the Davidon-Fletcher-Powell (DFP) method, where  $B(X, U)$  is a symmetric, positive definite matrix of the DFP algorithm.

The generalized objective function  $F(X, U)$  is defined for instance as an additive function:

$$F(X, U) = C(X) + \psi(X, U) \quad (7)$$

where  $C(X)$  is the ordinary objective function of the design process and  $\psi(X, U)$  is the additional penalty function, which includes some equations of the system (4) and can be defined for instance as:

$$\psi(X, U) = \frac{1}{\epsilon} \sum_{j=1}^M u_j \cdot g_j^2(X) \quad (8)$$

All control variables  $u_j$  are the functions of the current point of the design process. The total number of the different design trajectories, which are produced inside the same optimization procedure, is practically infinite. Among all of these strategies one or few optimal strategies exist that achieve the design objectives for the minimal computer time. Therefore, the problem of the optimal design strategy search is formulated as the typical problem for the functional minimization of the control theory. The main problem of this definition is unknown optimal dependencies of all control functions  $u_j$ . The minimal-time problem for the system (3), (4) with non-continued control functions can be solved most adequately by means of Pontryagin maximum principle [7]. The direct application of this principle to the non-linear problem is very problematical but the approximate methods can be used to solve it [8]-[10]. The approximate approach to solve this problem, which is proposed in this paper, is based on an additional acceleration effect [11] and on the behavior analysis of the special type of the Lyapunov function of the design process. Some significant characteristics of the acceleration effect of the design process were analyzed in the next section.

### 3. MAIN FEATURES OF THE DESIGN ACCELERATION EFFECT

The preliminary analysis of an additional acceleration effect of the design process was done in the paper [11]. This effect appears for all analyzed circuits and serves later on as the basis for the time-optimal algorithm construction.

Two-dimensional analysis has been done for a simplest electronic circuit with the topology, which is shown in Fig. 1.

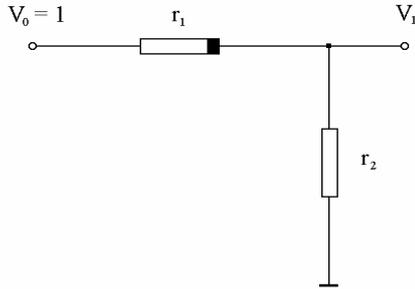


Figure 1. Topology of a simplest electronic circuit.

The element  $r_1$  has a non-linear dependency in general case:  $r = r_{10} + b_n \cdot V_1^2$ . The vector  $X$  of the state variables has two components  $X=(x_1, x_2)$ , where  $x_1$  is the independent parameter, ( $x_2 \equiv V_1$ ). The objective function is defined by the formula  $C(X) = (x_2 - k_V)^2$ , where  $k_V$  has the fixed value. There is only two coordinate of the vector  $X$  and only one control function  $u_1$  in this case. We have only two design strategies with the fixed value of the function  $u_1$ , for  $u_1=0$  (traditional design strategy) and for  $u_1=1$  (modified traditional design strategy). The design trajectory for this example is the curve in two-dimensional space. The main result of the preliminary analysis [11] is that: the behavior of the trajectories which corresponds to the traditional and modified traditional design strategies is very different and the behavior of the trajectories that corresponds to the modified traditional design strategies strongly depends on the start point of the design process. The trajectories, which correspond to the initial vector  $X_{in}$  with the components (1,-1) and for three different values of the non-linearity parameter  $b_n$  ( $10^{-5}$ , 1.0, 5.0) are presented in Fig. 2 (a), (b), (c) for the gradient optimization method. The trajectories that correspond to the traditional design strategy (solid line) are very different from the modified traditional strategy. For this last strategy the first part of the trajectory lies in a physically unreal sub-space ( $x_2 < 0$ ) and the second part lies in a real sub-space ( $x_2 > 0$ ). Moreover, it is very important to note that the movement along the trajectory is very fast from the start point  $S$  to the point  $R$ . On the other hand the movement is by far slower from the point  $R$  to the finish point  $F$ . It is very important that trajectories, which correspond to the traditional and the modified traditional strategies draw to the finish point  $F$  from the opposite directions. The unique possibility to accelerate the design process is created when the switching point of the control function  $u_1$  lies in the

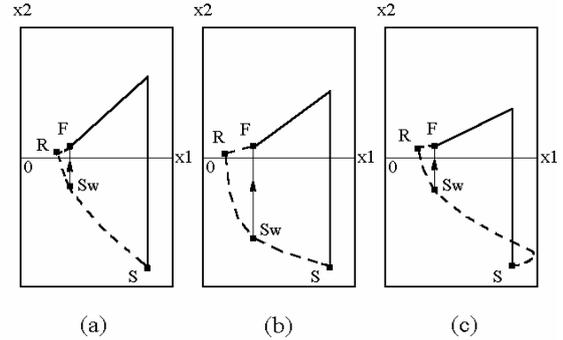


Figure 2. Trajectories for the traditional strategy (solid line) and for the modified traditional strategy (dash line) for  $X_{in}=(1,-1)$ . a)  $b_n=10^{-5}$ ; b)  $b_n=1.0$ ; c)  $b_n=5.0$ .

point, which is the projection of the finish point  $F$  to the modified traditional strategy trajectory, which lies in unreal sub-space. This is the point  $Sw$ . The optimal trajectory has two parts in this case. The first part corresponds to the curve  $S - Sw$ . During the movement along this curve the control function  $u_1$  is equal to 1. The control function  $u_1$  at the time moment, which corresponds to the point  $Sw$  changes the value to 0. At this moment the jump is realized from the point  $Sw$  to the finish point  $F$  or very near to the point  $F$  (it depends on the calculate step). Therefore a great acceleration of the design process takes place. This acceleration effect is observed for all values of the non-linearity parameter  $b_n$ . This effect is observed for the  $N$ -dimensional examples too. However, in this case a trajectory line of the design process lies in  $N$ -dimensional design space and we need to analyze different projections of  $N$ -dimensional curves.

The five-dimensional problem is discussed below for the circuit with the topology, which is shown in Fig. 3. This is a non-linear circuit that has three admittance  $y_1, y_2, y_3$  as independent parameters, ( $K=3$ ) and two node voltages  $V_1, V_2$  as dependent parameters, ( $M=2$ ).

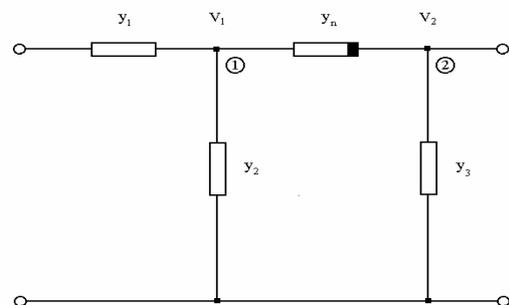


Figure 3. Circuit topology for  $K=3, M=2$ .

Non-linear element has dependency by the law:  $y_n = a_n + b_n \cdot (V_1 - V_2)^2$ . The vector  $X$  has five components  $X = (x_1, x_2, x_3, x_4, x_5)$  where  $x_1^2 \equiv y_1$ ,  $x_2^2 \equiv y_2$ ,  $x_3^2 \equiv y_3$ ,  $x_4 \equiv V_1$ ,  $x_5 \equiv V_2$ . The objective function  $C(X)$  has been determined as the sum of the squared differences between beforehand-defined values and current values of the nodal voltages for two nodes with additional inequalities for some circuit elements. However, it can be noted that the additional acceleration effect appears for the different types of the objective function. The data of the complete set of design strategy with constant value of the control function vector  $U$  and positive components of the initial vector  $X_{in}$  are presented in Table 1 for three different optimization procedures.

TABLE 1  
COMPLETE SET OF DESIGN STRATEGIES FOR THE INITIAL VECTOR  $X_{in} = (1,1,1,1,1)$ .

N	Control functions vector $U(u_1, u_2)$	Gradient method		Newton method		DFP method	
		Iterations number	Total design time (sec)	Iterations number	Total design time (sec)	Iterations number	Total design time (sec)
1	(00)	16	0.0243	7	0.0396	8	0.0241
2	(01)	51	0.0238	9	0.0251	10	0.0107
3	(10)	60	0.0448	8	0.0329	21	0.0331
4	(11)	68	0.0217	11	0.0231	23	0.0198

All these strategies are not time-optimal and the optimal design strategies for all optimization methods were found by means of the additional analysis. The results of this analysis are given in Table 2 for the non-linearity parameters  $b_n = 1.0$  and for two values of the initial vector  $X_{in} = (1,1,1,1,1)$  and  $X_{in} = (1,1,1,1,-1)$ .

TABLE 2  
DATA OF THE OPTIMAL DESIGN STRATEGY FOR TWO VALUES OF THE VECTOR  $X_{in} = (1,1,1,1,1)$ ,  $X_{in} = (1,1,1,1,-1)$

N	Method	Initial co-ordinate vector $X_{in}$	Optimal control functions vector $U(u_1, u_2)$	Iterations number	Switching points	Total design time (sec)
1	Gradient method	(1,1,1,1,1)	(10); (11)	39	11	0.0141
		(1,1,1,1,-1)	(11); (00); (11)	16	2; 3	0.0063
2	Newton method	(1,1,1,1,1)	(11); (10)	7	3	0.0228
		(1,1,1,1,-1)	(10); (00); (01)	5	1; 2	0.0181
3	DFP method	(1,1,1,1,1)	(01); (11)	10	9	0.0105
		(1,1,1,1,-1)	(11); (01)	7	2	0.0071

These results correspond to the analysis of the previous example. The optimal control functions and the optimal behavior of the design trajectories were obtained on the basis of some approximate methods of the optimal control theory [8]-[10]. The computer time gain of the optimal design strategy with respect to the traditional strategy is equal to 1.73, 1.74, and 2.3 for the gradient method, Newton method and DFP method respectively and for the first value of the initial vector  $X_{in}$ . An additional

acceleration effect is displayed in case when the initial vector  $X_{in}$  is equal to one of the two possible values: (1,1,1,1,-1) or (1,1,1,-1,-1). More effect is observed for the first value. This effect appears due to the trajectory jump, similar to the two-dimensional problem. However, in this case we have the five-dimensional space problem and the trajectory behavior is more complicated. The computer time gain in this case is equal to 3.85, 2.19, and 3.41 for three above mentioned optimization methods. So, in this case we have an additional time gain of 123%, 26%, and 48% for three different methods.

In general case, we have  $N$ -dimensional design problem. However, all specific features of the additional design acceleration, as a necessary trajectory jump, and a time gain are revealed again. The potential computer time gain of the optimum design strategy without and with an additional acceleration as the function of the dependent parameter number  $M$  is presented in Fig. 4 (a), (b) for three different optimization procedures. Fig. 4 (a) corresponds to the time gain without an additional acceleration effect when the initial value of the state variables are positive and Fig. 4 (b) corresponds to the time gain with an additional acceleration effect when the initial value of some state variables are negative.

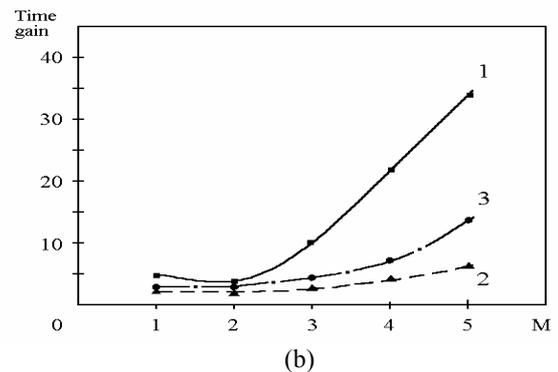
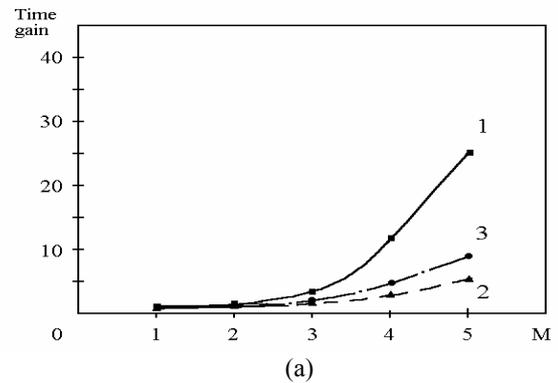


Figure 4. Optimal strategy potential computer time gain. 1-Gradient method, 2-Newton method, 3-DFP method. (a) without an additional acceleration effect; (b) with an additional acceleration effect.

The circuit topology for the different node number  $M$  has been taken from the paper [6]. The comparison of the curves of the figures 4 (a) and 4 (b) demonstrates that the additional acceleration effect is displayed for all analyzed examples and gives an additional time gain from 20% to 180% depending on the problem dimension and optimization method.

The active circuit analysis gives similar results. In Fig. 5 there is a circuit of the transistor amplifier that consists of three transistor cells.

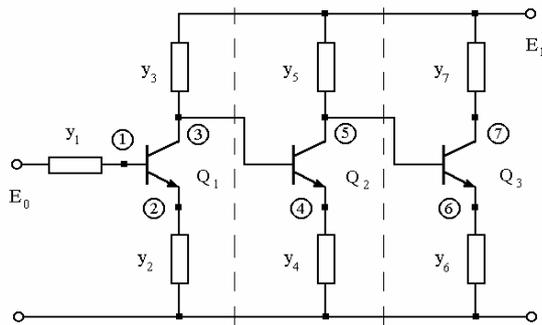


Figure 5. Circuit topology for three-transistor cell amplifier.

The one, two, and three transistor cell circuits were analyzed separately. The circuit includes three nodes ( $M=3$ ) for the first case. The second circuit includes two transistor cells and has five nodes ( $M=5$ ). The last case includes the full circuit of the Fig.5 with three transistors and seven nodes ( $M=7$ ).

The potential computer time gain of the optimal design strategy with an additional acceleration as the function of the transistor cell number  $N_{TR}$  is presented in Fig. 6 for two different optimization procedures (gradient method and DFP method).

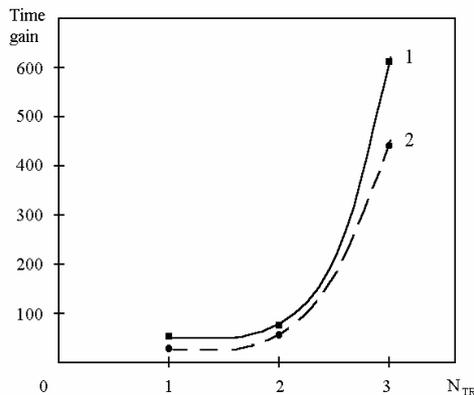


Figure 6. Optimal strategy computer time gain of the active circuits with an additional acceleration effect. 1-Gradient method, 2-DFP method.

The additional acceleration is observed when some components of the initial vector  $X_{in}$  are negative. However in this case the analysis is more complicated because the trajectory design line not always exists due to the specific current dependency of the transistor junctions. The additional time gain due to the acceleration effect is changed from 30% to 125% depending on the node number and the optimization method. The trajectory behavior near the finish point has a grate influence to the acceleration effect quantitative value. The complex behavior of the trajectories can complicate the acceleration effect achievement because there is more than one jump required in this case. Nevertheless the total computer time gain of the optimal strategy for the last example (three transistor cells circuit with 7 nodes and 14 variables) due to the acceleration effect is equal to 620 for the gradient optimization method and 477 for the DFP method.

#### 4. INITIAL POINT SELECTION

We start the analysis of the initial point selection and the influence of this initial point to the qualitative and quantitative characteristics of the acceleration effect from the simplest circuit of the Fig. 1 with non-linearity coefficient  $b_n = 1.0$ . The family of the curves, which correspond to the modified traditional design strategy ( $u=1$ ) and the negative initial value of the second coordinate ( $x_2 < 0$ ) of the vector  $X$  is shown in Fig. 7 for the 2-D phase space.

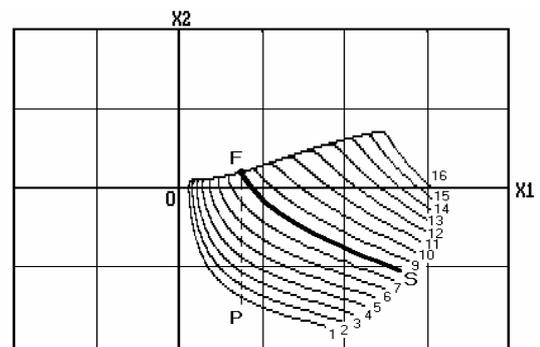


Figure 7. Trajectories of the modified traditional strategy for the different start points with the negative coordinate  $x_2$ .

These curves have different start points but have the same final point F. The start points were selected on the circle arc and have the different initial coordinates. The special curve S-F, which is marked by thick line, is the separating curve. This curve separates the trajectories that are the candidates for the acceleration effect achievement (all curves that lie under the curve S-F), and the trajectories that cannot produce the acceleration effect (curves that lie

over the curve S-F). It is clear that the projections of the finish point F to all curves of the first group define the switching point of the optimal trajectory, which produces the acceleration effect. These projections are defined by the intersection of the vertical line F-P and the corresponding curve of the first group. All curves of the first group (1-7) approach to the finish point F from the left side, and on the contrary, all curves of the second group (9-16) approach to the finish point from the right side. The S-F is the unique curve that goes to the finish point directly without any additional movement near the point F. The comparison of the computer time for all curves of the Fig. 7 shows the advantage of the curve S-F. The relative computer time  $\tau$  for all trajectories of the Fig. 7 is shown in Fig. 8 as the function of the curve number  $n$ .

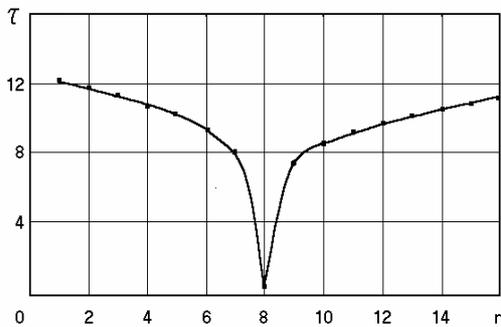


Figure 8. Relative computer time  $\tau$  as the function of the curve number  $n$ .

The separating curve S-F (number 8) has the minimal computer time among all of the trajectories. At the same time this curve does not produce the time-optimal design strategy. First of all nobody can guess the start point that lies in this separating curve exactly. Other consideration is more important. This curve cannot be used as the basis for the time-optimal trajectory construction because the projection of the point F to this curve is the same point F, but the movement slows down near this point along the all design trajectories. Only the curves that lie under the curve S-F serve as the first part of the time-optimal trajectory with the following jump to the point F. It is interesting to compare the computer time of the different time-optimal trajectories that have various start points. The relative computer time  $\tau$  of the optimal trajectories with acceleration effect (on the basis of the curves 1-7, Fig. 7) as the function of the curve number  $n$  is shown in Fig. 9. The curves 9-16 can be optimized too by means of the total trajectory compose on some parts with the different control function value (1 or 0). However we have the time reduction about 10-15% only in this case. There is no sharp acceleration for all of these curves. The Fig. 9

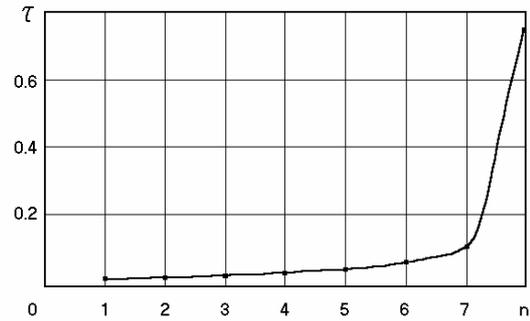


Figure 9. Relative computer time  $\tau$  of the optimal trajectories with acceleration effect as the function of the curve number  $n$ .

shows that the total computer time increases when the start point approaches to the curve S-F, and on the contrary, the more acceleration can be obtained if we select the start point further from the curve S-F (from curve 7 to curve 1). All these considerations are correct when the principal equations (1)-(2) are integrated by the relatively small step. In this case the continuity approach is used. On the other hand, we can achieve the switching point of the optimal trajectory by one or few steps in the discrete approach and the computer time of the optimal trajectory practically does not depend from the initial point. This dependency like the curve in Fig.7 for  $n \in [1,7]$ .

By this analysis we can consider that numerical results of the section 3 are not unique. These are the typical results and they were obtained for the initial points, which correspond to the curves that lie under the separating curve. By means of the discrete approach we can control some first steps of the design procedure to obtain the minimal steps to reach the switching point. In this sense the computer time of the optimal strategy depends insignificantly on the initial point selection because the switching point can be reached for one or few steps. The independence of the optimal computer time and the time gain from the initial point selection implies however that these initial points lie on the curves of the first group, i.e. under the separating curve.

In general we can conclude that the acceleration effect appears when the start point is selected under the separating line and this effect has the better characteristics when the start point lies far from this separating line. So, the start point selection with at least one negative initial coordinate of the vector  $X$  and the value of this coordinate that provides the start point position under the separating line are the sufficient conditions for the acceleration effect appearance. All these conclusions are correct for the  $N$ -dimensional problem too. We need to analyze the different projections of the  $N$ -dimensional curve in this case. The  $N$ -dimensional problem solution gets complicated by a large number of the different admissible

trajectories and a large number of the different trajectory projections. In this case we need to choose the most perspective trajectories and analyze them. In this paper it was done by the careful analysis of all possibilities. The total number of the different design trajectories with the fixed control function vector  $U$  for the  $M$ -node circuit is equal to  $2^M$ , as shown in [6]. This set of the various trajectories can be divided in two different subsets. The first subset consists of the trajectories that are similar to the traditional design strategy trajectory, as for example the solid-line curves of the Fig. 2. The second subset consists of the trajectories that are similar to the modified traditional strategy trajectories as the dash-line curves of the Fig. 2. In this case the trajectories of the second group serve as the candidates for the first part of the optimal trajectory and the first group trajectories serve as the candidates to the jump produce. Two of these main steps together with the following different trajectory adjustment make up the essence of the optimal algorithm construction. By the experience in section 3 we can decide that not all of the feasible projections are important to the acceleration effect obtained. First of all the admittance-voltage two-dimensional projections are more important. Among all of this type projections are of great importance those variables that are included in the objective function formula. By this preliminary selection we can reduce the number of the more perspective candidates for the time-optimal algorithm elaboration. This problem final solution will be based on the optimal algorithm intrinsic structure. However, the results obtained in this paper serve as the next step on the way of this problem solution. Now it is clear that the optimal algorithm must include the special conditions to the acceleration effect reach. On the other hand the problem of the concrete trajectories selection from a large set of the different trajectories and the switching point position determination can be solved inside the time-optimal algorithm.

The sufficient conditions for the acceleration effect existence were defined above. However, these conditions do not define the suitable in practice value for the optimal start-point because nobody knows the separating line position in advance. We obtained the separating line position after the design problem analysis, but the real system design means the selection of the optimal start point beforehand and the movement along the optimal trajectory. All these problems are the essence of the optimal design algorithm construction. This problem does not have the exact solution until the moment; however the acceleration effect existence and its principal characteristics serve as the foundation for the optimal algorithm search. As to optimal start point choice, this problem can be solved by means of the negative coordinate selection sufficiently large in the absolute value, which ensure the acceleration effect existence.

## 5. ON OPTIMAL ALGORITHM STRUCTURE

On the basis of the analysis in previous section we can conclude that the time-optimal algorithm has one or some switching points where the switching realize from like modified traditional strategy to like traditional strategy with an additional adjusting. At least one negative component of the start value of the vector  $X$  is needed for the optimal trajectory obtained.

The main problem of the time-optimal algorithm construction is unknown sequence of the switching points during the design process. We need to define special criteria that permits realize the optimal or quasi-optimal algorithm by means of the optimal switching points searching. In this paper we propose to use a Lyapunov function of the design process for the optimal algorithm structure revelation, in particular for the optimal switching points searching. There is a freedom of the Lyapunov function choice because of a non-unique form of this function. Let define the Lyapunov function of the design process as:

$$V(Y,U) = \sum_i \left( \frac{\partial F(Y,U)}{\partial x_i} \right)^2 \quad (9)$$

where  $F(Y,U)$  is the generalized objective function of the optimization procedure. This form holds all of the necessary characteristics of the standard Lyapunov function definition. It is supposed that the vector  $Y$  is defined as the difference between two vectors  $X$  and  $A$ , where  $A$  is the stationary point of the design process (the final point). First of all the function (9) can be used for the stability analysis of the design process. In this context this function is used for the analysis of the design trajectories behavior with the different switching points. We can define now the system design process as a transition process that can provide the stationary point (optimal point of the design procedure) during some time. The problem of the time-optimal design algorithm construction is the problem of the transition process searching with the minimal transition time. There is a well-known idea [12]-[13] to minimize the transition process time by means of the special choice of the right hand part of the principal system of equations, in our case these are the functions  $f_i(X,U)$ . It is necessary to change the functions  $f_i(X,U)$  by means of the control vector  $U$  selection to obtain the maximum speed of the Lyapunov function decreasing (the maximum of  $-dV/dt$ ) at each point of the process. Unfortunately the direct using of this idea is not serves well for the time-optimal design algorithm construction. It occurs because the change of the design strategy produces not only continuous design trajectories (when we change the

strategy  $u=0$  to the strategy  $u=1$  for the circuit in Fig. 1 for instance) but non-continuous trajectories too (the changing from  $u=1$  to  $u=0$ ). Non-continuous trajectories had never been appeared in control theory for the objects that described by differential equations, even for the equations with variable structure, but this is the ordinary case for the design process on the basis of the described design theory. In this case we need to correct the idea of the value  $-dV/dt$  maximize at each point of the design process. We define another principle: it is necessary to obtain the maximum speed of the Lyapunov function decreasing for that trajectory part that lies after the switching point. In this case the trajectories with the different switching points are compared to obtain the maximum value of  $-dV/dt$ . Technically this idea is realized by comparing some probes with the different switching points and selecting the one of them that provides the maximum of  $-dV/dt$  after the switching. Fig. 10 shows the behavior of the Lyapunov function time derivative for three consecutive neighbor-switching points 1, 2, 3 and for five consecutive displacements of these points during the design trajectories around the optimal position for the circuit of Fig. 5. Fig. 10 (a), (b) correspond to the switching points those lie before the optimal position. We can see in this case that the behavior of the curve 2 is better than curve 1 because  $-dV/dt_2 > -dV/dt_1$  and the curve 3 is better than the curve 2 because  $-dV/dt_3 > -dV/dt_2$ . Fig. 10 (c) corresponds to the optimal position of the switching point. The point 2 of Fig. 10 (c) is the optimal because the left neighbor and the right neighbor are worse. Fig. 10 (d), (e) correspond to the switching points that lie after the optimal position.

Analysis of these results and the data, which correspond to the other circuit types, shows that the idea of use of the Lyapunov function time derivative serves well in combination with the additional acceleration effect. In this case the optimal position of the switching point can be found. These ideas can make up the basis for the accurate construction of the time-optimal design algorithm.

It is clear that we need to calculate some additional probes during the optimal switching point position searching. This is the necessary pay for the optimal trajectory structure revelation. In this case we cannot obtain the time gain, which corresponds to the time-optimal design strategy because we need to search the switching point optimal position during the design process. The time waste make up until 100% from the optimal design strategy time. It means that really we cannot obtain the time gain 620 or 477 for the circuit in Fig. 5, but a two times less (300 or 250). However these values are significant too and the total design time reduction is the sufficient basis for the new design methodology development.

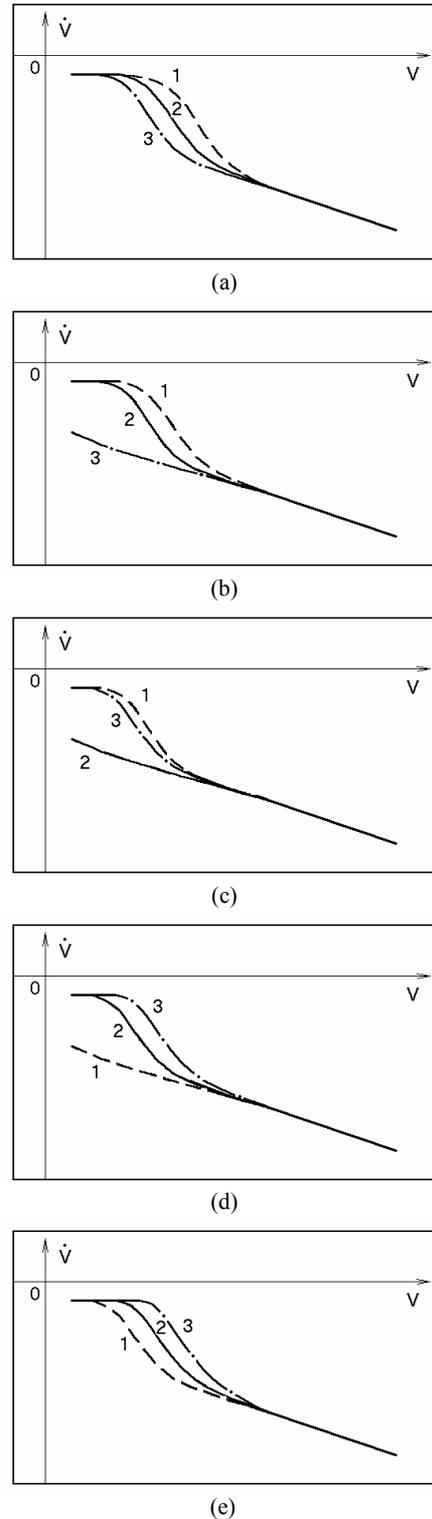


Figure 10. Time derivative of Lyapunov function behavior for three switching points 1,2,3 of the consecutive integration steps before (a), (b), in (c) and after (d), (e) the optimal point.

## 6. CONCLUSIONS

The traditional approach to the analog system design is not time-optimal. The problem of the time-optimal algorithm construction can be solved as the functional optimization problem of the control theory. The analysis of the different electronic systems gives the possibility to conclude that the potential computer time gain of the time-optimal design strategy increases when the size and complexity of the system increase. The additional acceleration effect of the system design process was discovered by means of the variation of the initial value of the state variables and the special control functions. This effect exists owing to the very different behavior of the design trajectories that have various control functions and different start points of the design space. The initial point selection permits obtain acceleration effect with a great probability. This effect reduces the total computer time additionally and serves as the basis for the optimal or quasi-optimal algorithm construction. The optimal position of the necessary switching points can be obtained on the basis Lyapunov function. The minimization of the time derivative of this function serves as the principal criterion for the optimal switching point's definition. Thus the combination of the acceleration effect with the optimal switching points serve as the principal ideas for the quasi-optimal algorithm construction.

## 7. REFERENCES

- [1] Osterby O., and Z. Zlatev, *Direct Methods for Sparse Matrices*, Springer-Verlag, N.Y., 1983.
- [2] A. George, "On Block Elimination for Sparse Linear Systems", *SIAM J. Numer. Anal.* vol. 11, no.3, 1984, pp. 585-603.
- [3] F.F. Wu, "Solution of Large-Scale Networks by Tearing", *IEEE Trans. Circuits Syst.*, vol. CAS-23, no. 12, 1976, pp. 706-713.
- [4] A. Sangiovanni-Vincentelli, L.K. Chen, and L.O. Chua, "An Efficient Cluster Algorithm for Tearing Large-Scale Networks", *IEEE Trans. Circuits Syst.*, vol. CAS-24, no. 12, 1977, pp. 709-717.
- [5] A. Zemliak, "System Design Problem Formulation by Control Theory", *Proc. of the IEEE ISCAS 2001*, Sydney, Australia, 2001, Vol. 5, pp. 5-8.
- [6] A. Zemliak, "Analog System Design Problem Formulation by Optimum Control Theory", *IEICE Trans. Fundamentals*, vol. E84-A, No 8, 2001, pp.2029-2041.
- [7] L.S.Pontryagin, V.G.Boltyanskii, R.V.Gamkrelidze, E.F. Mishchenko, *The Mathematical Theory of Optimal Processes*, Interscience Publishers Inc., New York, 1962.
- [8] Krotov V.F, *Global Methods in Optimal Control Theory*, Marcel Dekker, Inc., N.Y., 1996.
- [9] Sepulchre R., M. Jankovic, and P.V. Kokotovic, *Constructive Nonlinear Control*, Springer-Verlag, N.Y., 1997.
- [10] Pytlak R., *Numerical Methods for Optimal Control Problems with State Constraints*, Springer-Verlag, Berlin, 1999.
- [11] A. Zemliak, "Acceleration Effect of System Design Process", *IEICE Trans. Fundamentals*, vol. E85-A, No. 7, 2002, pp.1751-1759.
- [12] Barbashin E. A. *Introduction to the Stability Theory*, Nauka, Moscow, 1967.
- [13] Rouche N., P. Habets, and M. Laloy, *Stability Theory by Liapunov's Direct Method*, Springer-Verlag, N.Y, 1977.